Math 211 - Bonus Exercise 9 (please discuss on Forum)

1) Prove that there are no simple groups of order 308 and 364.

2) Using the Sylow theorems and Exercise 6 on this week's sheet to prove the following generalization of Lemma 14 in the notes:

Consider a subgroup $H \leq G$ of a finite group G, ***not necessarily normal****. Then for any prime p,* ***there exists*** *a Sylow p-subgroup of G whose intersection with H will be a Sylow p-subgroup of H.*

3) Describe the Sylow p-subgroups of the dihedral group for odd p.

4) Find a Sylow p-subgroup of the group $GL_2(\mathbb{F}_p)$ of invertible 2×2 matrices with entries in the field $\mathbb{F}_p$, with respect to matrix multiplication. We're not studying fields in this class, but you encountered this notion in Algebraic Structures: all this means is that the group $GL_2(\mathbb{F}_p)$ consists of invertible 2×2 matrices whose coefficients are taken to be residues mod p.